Design of Forming Shoulders with Complex Cross-sections

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Vertical ‘form, fill and seal’ machines are used to produce bags for packing particulate or multiple objects. In operation, film is drawn over a forming shoulder and the good design of the surfaces of the shoulder is vital to the successful operation. This paper reviews underlying geometrical definition for the shoulder, corresponding to a filling tube with circular cross-section. In practice, such cross-sections occur frequently, but other variant shapes are becoming increasingly common. A method is discussed and demonstrated for extending the approach to allow tubes formed from circular arcs and straight line segments to be handled.

INTRODUCTION

Very few goods are now bought that are not sold in some sort of presentation or protective packaging. The largest and most visible sector of the packaging industry is that of fast-moving consumer goods (FMCG), which includes, for example, food products and pharmaceutical products. In the food products sector, packaging design and material not only represent an investment in physical and environmental protection but are also an integral and particularly important part of sales and marketing. In today’s consumer-driven markets the graphical design and form of the pack are very important for brand recognition, product differentiation and, ultimately, commercial success. Of particular interest are bags or pouches with square or rectangular cross-section. These have benefits for transporta-

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opment of new processes, machine systems and components.

This paper deals with a method to support the design of complex tooling for a high-speed packaging process. This process involves the production of a bag or pouch from a reel of packaging material, which is then filled with product and sealed. This process is shown in Figure 1 and is more widely known as a ‘form, fill and seal operation’. Such a process is commonly used to package products such as crisps, snacks and pasta. It has been adapted to produce a variety of different packages, such as packs with bottom gussets or flat bottoms.3 The critical material handling element is the forming shoulder. Its function is to guide the incoming flat web of material (at high speed) into a bag or pouch. As previously discussed, many manufacturers of FMCG are now requiring bags or pouches of more complex cross-sections. This may include approximately square cross-sections or sections that are re-entrant or fluted. These demands add considerable additional complexity to the design and manufacture of an already intricate machine component (the forming shoulder). In particular, it is very difficult to create the former without a understanding of the surface geometry of the shoulder. The ability to achieve this is further frustrated by the current design and manufacturing processes employed by many machinery producers.

Current design practice is discussed in the next section and a numerical model of the geometry of the forming shoulder is formulated. This model is further extended to enable the design of forming shoulders with complex cross-sections, as well as shoulder geometry that can be matched to the properties of a particular material. The method is demonstrated and applied to the creation of a forming shoulder for the production of packs with an approximately square cross-section.

CURRENT DESIGN PRACTICE

This paper considers the process of the vertical form, fill and seal operation and the design of the key machine component, the forming shoulder. The function of the forming shoulder is to guide the material from a flat roll to a tubular shape. The edges of the film are brought together and sealed to form a tube into which the product can be inserted in measured quantities. The tube of packaging material is then cross-sealed to form a closed bag. The final seal also forms the base seal of the next bag. The successful creation of the pack is largely dependent upon the performance of the forming shoulder, which in turn is highly dependent upon the surface geometry of the shoulder.4

Although some attempts have been made at casting or moulding forming shoulders, the majority of formers are still constructed from a flat metal sheet cut into in two parts. For the purpose of this paper, these sections are referred to as ‘the tube’ and ‘the collar’, as shown in Figure 2. The tube portion of the forming shoulder has a circumference equal to twice the full bag width.

There are several approaches to defining the geometry of the shoulder.5–7 These methods start with a bending curve, as shown in Figure 3. The curve divides the plane into two parts. The part below the curve can be thought of as being rolled to form the shape of the tube (although this is not normally done in practice). The part above is wrapped in the other direction to form the collar of the shoulder (and doing this using sheet metal is one method of manufacture). Additionally, the upper piece may be split in two and a planar triangular insert added into the collar to aid the flow of the film from the rollers. In practice, the bending curve may be selected from a range of previously
used curves. A better understanding allows reliable shoulders to be produced, as well as variants on the basic shape.

The procedure reported in this paper extends this previous work and is consistent with the design method relating to machine–material interaction.4 The method for the creation of shoulders with complex cross-sections is discussed in the next sections and its application to a case study example described.

**SHOULDER GEOMETRY**

The geometry of the forming shoulder is essentially defined by the bending curve. As in Boersma and Molenaar,5 let \( \bar{r}(s) \) denote the bending curve when it is regarded as a planar curve. Once it has been wrapped into the third dimension, it is denoted by \( r(s) \). It is, of course, the same arc length, \( s \), in both cases.

The basic notation for the planar version of the bending curve is shown in Figure 3 and is discussed below. Note that the unit normal \( \bar{n} \) is inward-pointing. As the collar can be formed from the part of the plane above the curve, it is a developable surface (i.e. it is isomorphic to the plane) and is formed by generators which are straight lines lying in the surface. The surface is defined (and hence can be modelled and manufactured) once the positions of the generators are known.

In Figure 3, \( \bar{d} \) is a unit vector that maps to \( d \), which lies along the typical generator of the collar surface. This is a straight line lying in the surface. Following Boersma and Molenaar,5 in the plane the generator is:

\[
\bar{d}(s) = \cos \alpha(s) \bar{i}(s) - \sin \alpha(s) \bar{n}(s)
\]  

and for the surface:

\[
d(s) = \cos \alpha(s) t(s) - \sin \alpha(s) \left[ \cos \phi(s) n(s) + \sin \phi(s) b(s) \right]
\]  

Here \( \alpha \) is the angle between the generator and the tangent to the curve; this is the same in the planar and surface cases. Angle \( \phi \) determines how the generator moves away from the tangent direction once the third dimension is introduced. The generators are determined once \( \alpha \) and \( \phi \) are known.

The typical points in the plane and on the collar are as follows:

\[
\bar{S}(s, u) = \bar{r}(s) + u \bar{d}(s)
\]

\[
S(s, u) = r(s) + u d(s)
\]

The theory developed in Boersma and Molenaar5 derives the following expression for \( \phi \) and \( \alpha \) in terms of the curvature \( \bar{\kappa} \) of the planar bending curve and the curvature \( \kappa \) and the torsion \( \tau \) of the bending curve on the shoulder. These parameters can be determined using the Frenet–Serret formulae.8

\[
\kappa \cos \phi = \bar{\kappa}
\]
where is a general property for developable surfaces. One design approach is to take it to be parabolic. The collar surface is given by equation (3). Its partial derivatives are as follows:

\[
S_u = d = \cos \alpha t - \sin \alpha (\cos \phi n + \sin \phi b)
\]

\[
S_v = t + u d_v = [1 + u(\alpha - \alpha_0)]\sin \alpha t
+ u(\alpha - \alpha_0)\cos \alpha (\cos \phi n + \sin \phi b)
\]

The surface normal is in the direction of the vector product of these:

\[
S_u \times S_v = [\sin \alpha + u(\alpha - \alpha_0)](\cos \phi b - \sin \phi n)
\]

and hence a unit normal at a general position on the collar is as follows:

\[
\cos \phi b - \sin \phi n
\]

This is an important result, as the normal is not dependent upon the parameter. The direction of the normal to the collar surface lies in the same direction for all points along any generator (this is a general property for developable surfaces). The direction is determined by what happens at the point on the bending curve from which the generator emanates.

**BENDING CURVE**

The ideas of the last section are valid for any (reasonable) bending curve and tube crosssection. It is important to understand the form of the curve in order to obtain a good shoulder. For completeness, in this section a specific form of the bending curve is here discussed. The tube is assumed to be a circular cylinder of radius \(R\), and this is extended in the next section.

The bending curve is described in its planar form as a curve \(z(v)\), where the parameter \(v\) runs between \(-\pi R\) and \(\pi R\). Thus, \(v\) corresponds to a circumferential position around the tube. The total circumference is \(2\pi R = 2w\), where \(w\) is half the width of the film (ignoring the overlap required for sealing).

In its planar state, the bending curve is roughly parabolic. One design approach is to take it to be exactly parabolic and use the following equation:

\[
z(v) = h[1 - (v/R)^2] \quad \text{for} \quad -\pi R \leq v \leq \pi R
\]

where \(h\) is the height of the top of the bending curve above its lowest point. One of the key design parameters is the ratio \(h/R\). In practice this normally lies between 2 and 5, with the lower values being the more common.

The above form of \(z\) is everywhere infinitely differentiable. In practice, packaging film is fed from a roll. This means that material starts as a flat plane and there is a need for a smooth transition between the plane and the collar. To this end, a triangular planar surface needs to be included in the collar at its highest point. The collar is formed from straight generators leading from the bending curve. If the curve is smooth, then the generators emanate at smoothly varying angles and the surface is continuous. However, if the curve has a suitable discontinuity at its highest point, then the generators on either side are in different directions and a planar triangle can be inserted into the collar. If the bending curve \(z(v)\) in its planar version is an even function, so that \(z(-v) = z(v)\), then the surface has symmetry. The appropriate discontinuity is one in which the third derivative of \(z\) is everywhere continuous, except at \(v = 0\). Let \(\beta\) be the angle between the generators at the highest point of the collar; this is also the angle at the apex of the inserted triangle, and is called the ‘opening angle’. It is given by the following, where it is assumed that the sign convention and choice of bending curve is such that \(z_{\text{evn}}(0+)\) is negative:

\[
\tan \left(\frac{1}{2} \beta\right) = \frac{2R^2 z_{\text{evn}}(0+)}{R^2 z_{\text{evn}}(0)+1}
\]

The following modified form of bending curve is introduced:

\[
z = z(v) = Rf(\xi)
\]

where \(\xi = v/R\) for \(-\pi \leq \xi \leq \pi\), and \(f(\xi)\) is the even function given by:

\[
f(\xi) = \begin{cases} 
  c_0 + c_2 \xi^2 + c_3 \xi^3 & \text{for} \quad \xi \geq 0 \\
  + c_4 \left(\cos \xi - 1 - \frac{1}{2} \xi^2\right) \\
  + c_5 \left(\sin \xi - \xi - \frac{1}{6} \xi^3\right) & \text{for} \quad \xi < 0 
\end{cases}
\]

There are five coefficients, \(c_0, c_2, c_3, c_4\) and \(c_5\), involved here and these are related by the assump-
tion that \( f(\pi) = 0 \). They can be determined (subject to certain restrictions to ensure that the collar is free of discontinuities) in terms of geometric parameters of the shoulder. Two of these parameters are the ratio \( h/R \) and the angle \( \beta \) of the triangular insert. The other two relate to the angle \( \theta \) between the normals to the collar and the tube at the front and back of the shoulder. In the general position, this angle is given by:

\[
\cos \theta = \left( \frac{R^2 z_v^2 - z_v^2 - 1}{R^2 z_v^2 + z_v^2 + 1} \right)
\]

where the subscript \( v \) denotes partial differentiation with respect to \( v \).

Figure 4 gives an example of bending curve and collar surface generated using the above approach. The surface lines running away from the curve are the generators. A planar triangular insert is present in the collar at the top of the curve. The coefficients for this surface are: \( c_0 = 4.0000, c_3 = -0.2071, c_4 = -0.0976, c_5 = -0.1408, c_6 = 0.7327 \). These correspond to \( (h/R) = 4, \beta = 90^\circ \), and the values of \( \theta \) at the highest and lowest points of the curve being 45° and 10°, respectively.

**NON-CIRCULAR SECTIONS**

The form of tube assumed in the previous section is circular. This is a common requirement for forming shoulders. However, some variations from this basic shape are needed from time to time. These include, for example, tubes with (roughly) square or rectangular section. One technique is based on the intersection of a ‘super-ellipse’ and a developable shape consisting of two cones with a flat plane in between. A ‘super-ellipse’ is a curve with the equation:

\[
\left( \frac{x}{a} \right)^n + \left( \frac{y}{b} \right)^n = 1
\]

where \( n \) is an integer. For high values of \( n \), the shape becomes approximately rectangular.

One problem with tubes generated using super-ellipses is that the curves in the crosssection are no longer circular arcs and thus are difficult to manufacture using traditional techniques. The aim here is to show how the approach of the previous sections can be extended to allow the introduction of straight line segments into the tube cross-section and the bending curve.

The enhanced procedure is to divide the shoulder based on a circular tube along generators emanating from selected points on the bending curve, and then insert additional regions to fill in the gaps in the collar and tube. The result is a forming shoulder with a tube that has flat sides and corners that are circular arcs. These have radii equal to that of the original tube. The collar inherits properties from the original circular shoulder. In particular, shoulders of the enhanced form can be created with a given opening angle, \( \beta \), and front and back angles.

Figure 5 shows the procedure applied to divide a shoulder into four quadrants along appropriate generators. These are for points on the tube at 90° intervals. The gaps between the generators then need to be filled in with planar regions. The result is a shoulder with a rectangular tube with rounded corners. This case is now considered further; it is easily adapted for other cases.

The modified tube cross-section is shown in the upper part of Figure 6. If the lengths of the inserted straight line portions are 2a and 2b, then the total circumference becomes 2w, where \( w = \pi R + 2a + 2b \) is the semi-bag circumference. Suppose that \( z(v) \)
for $-\pi R \leq v \leq \pi R$ is the original planar bending curve, and that $Z(v)$ for $-\pi R + 2a + 2b \leq v \leq (\pi R + 2a + 2b)$ is the enhanced one. The latter is shown in the lower part of Figure 6 and is determined by the property that $Z(-v) = Z(v)$ and the following relations. It is obtained by splitting the original bending curve at the appropriate $v$ values and separating the parts by the appropriate amounts in the $v$-direction. They are also moved in the $z$-direction, so that the tangent at the end of one side of the split is the same as that at the end of the other side.

$$Z(v) = \begin{cases} 
    z_1 & \text{for } 0 \leq v \leq v_1 \\
    z(v - v_1) + z_2 - z_3 + z_4 & \text{for } v_1 \leq v \leq v_2 \\
    z_3 \left( \frac{v_3 - v}{v_3 - v_2} \right) (z_2 - z_3) & \text{for } v_2 \leq v \leq v_3 \\
    z(v - v_3 + v_2 - v_1) + z_4 & \text{for } v_3 \leq v \leq v_4 \\
    \left( \frac{v_5 - v}{v_5 - v_4} \right) z_4 & \text{for } v_4 \leq v \leq v_5 
\end{cases}$$

where:

$v_1 = a$
$v_2 = v_1 + \frac{1}{2} \pi R$
$v_3 = v_2 + 2b$
$v_4 = v_3 + \frac{1}{2} \pi R$
$v_5 = v_4 + b$

and:

$z_4 = (v_5 - v_4)z'(\pi R)$
$z_3 = z_4 + z(1) + \frac{1}{2} \pi R$
$z_2 = z_3 + (v_3 - v_2)z'(\frac{1}{2} \pi R)$
$z_1 = z_2 + z(0) - \frac{1}{2} \pi R$

It remains to fill in the gaps in collar surface. Use is made of the observation made at the end of the section on shoulder geometry that the normal to
the collar surface is always in the same direction along any generator. This means that the gaps can be filled with planar regions. The plane is that perpendicular to the surface normal along the generator where the split is made, i.e. it is the extension of the common tangent plane on either side of the split. Since this, in particular, contains the tangent to the original bending curve, the result agrees with the modified bending curve.

The resultant completed surface for the particular example is shown in Figure 7, and as a shaded image in Figure 8.

In the section on Current design practice, the need to consider machine–material interaction and match shoulder configuration to the properties of a given material is discussed. For the method described here, the shoulder configuration (principally the height/radius ratio, \( h/R \)) is determined by considering the shoulder for the production of circular bags of circumference \( 2\pi R \) with the given material. This enables a shoulder geometry and configuration to be determined through consideration of all the necessary design constraints, including compactness and tracking.

**CONCLUSIONS**

The mathematical description of the geometry of the surfaces involved in the shoulder of a form, fill and seal machine is complicated. It is necessary to have an understanding of it in order to appreciate how the design variables interact. It is also required if improved means of manufacturing shoulders are to be obtained.

The basic theory is for a tube with a circular cross-section. In practice, other cross-sectional shapes may be required. The geometry of the collar surface is such that it is defined by straight line generators emerging from the bending curve. Along each generator, the surface normal is in a constant direction. This means that the surface can be split along a generator, the parts separated and the gap filled with a planar region. This allows collars to be defined, corresponding to tube cross-sections composed of circular arcs joined by tangential straight lines.

An example of this approach has been illustrated. While this is for a tube which is rectangular with rounded corners, the approach can be applied to any cross-section that can be derived from a splitting of the original circular section.
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